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## Constraining the Proton's Gluon Density by Inclusive Charm Electroproduction at HERA\*

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### Abstract

We analyze the capability of charm production in deep-inelastic  $ep$  scattering at HERA to constrain the gluon distribution  $g(y, \mu^2)$  of the proton. The dependence of the theoretical predictions for the charm structure function  $F_2^c$  on the mass factorization scale  $\mu$  and the charm mass is investigated.  $F_2^c$  seems to be well suited for a rather clean and local gluon measurement at small momentum fractions  $y$ .

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# CONSTRAINING THE PROTON'S GLUON DENSITY BY INCLUSIVE CHARM ELECTROPRODUCTION AT HERA

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We analyze the capability of charm production in deep-inelastic  $ep$  scattering at HERA to constrain the gluon distribution  $g(y, \mu^2)$  of the proton. The dependence of the theoretical predictions for the charm structure function  $F_2^c$  on the mass factorization scale  $\mu$  and the charm mass is investigated.  $F_2^c$  seems to be well suited for a rather clean and local gluon measurement at small momentum fractions  $y$ .

Recent measurements of the structure function  $F_2^{ep}$  at HERA rather directly determine the (light) quark densities in the proton at very small momentum fractions  $y$ . On the other hand, extractions of the gluon distribution from  $F_2$  scaling violations are more indirect and require assumptions on the evolution kernels at small  $y$ , e.g. of the validity of the usual NLO evolution equations, which themselves are a matter of investigation presently. An obvious candidate for a more direct gluon determination is the longitudinal structure function  $F_L$ , but in practice this quantity is not easily measured sufficiently accurate over a wide kinematical range. In this context it is interesting to recall that  $F_2$  itself is expected to contain an appreciable part directly sensitive to the gluon density at small Bjorken- $x$ , namely its charm production contribution  $F_2^{c\,1}$ . In the following we discuss some theoretical aspects of this observable relevant to the determination of  $g(y, \mu^2)$ ; for the experimental status at HERA see ref. <sup>2</sup>.

In next-to-leading order (NLO) perturbative QCD, the electromagnetic (exchange of one photon with virtuality  $Q^2$ ) structure function  $F_2^c$  reads <sup>3</sup>

$$F_2^c(x, Q^2) = \frac{Q^2 \alpha_S(\mu^2)}{4\pi^2 m_c^2} \int_{ax}^1 \frac{dy}{y} yg(y, \mu^2) e_c^2 \left\{ c_{2,g}^{(0)} + 4\pi\alpha_S(\mu^2) \left[ c_{2,g}^{(1)} + \bar{c}_{2,g}^{(1)} \ln \frac{\mu^2}{m_c^2} \right] \right\} \\ + \frac{Q^2 \alpha_S^2(\mu^2)}{\pi m_c^2} \sum_{q=u,d,s} \int_{ax}^1 \frac{dy}{y} y(q + \bar{q})(y, \mu^2) \left[ c_{2,q}^{(1)} + \bar{c}_{2,q}^{(1)} \ln \frac{\mu^2}{m_c^2} \right] . \quad (1)$$

Here  $m_c$  ( $e_c$ ) denotes the charm mass (charge), and  $\alpha_S$  is the strong coupling constant. The renormalization scale has been put equal to the ( $\overline{\text{MS}}$ ) mass factorization scale  $\mu$ . The lower limit of integration over the fractional initial-parton momentum  $y$  is given by  $y_{\min} = ax = (1 + 4m_c^2/Q^2)x$ , corresponding to the threshold  $\hat{s} = 4m_c^2$  of the partonic center-of-mass (c.m.) energy squared. The NLO coefficient functions  $c^{(1)}$  and  $\bar{c}^{(1)}$  have been calculated in ref. <sup>3</sup>, and a convenient parametrization of these results has been provided <sup>4</sup> in terms of

$$\xi = \frac{Q^2}{m_c^2} \quad , \quad \eta = \frac{\hat{s}}{4m_c^2} - 1 = \frac{\xi}{4} \left( \frac{y}{x} - 1 \right) - 1 \quad . \quad (2)$$

In leading order (LO),  $\mathcal{O}(\alpha_S)$ ,  $F_2^c$  is sensitive only to  $g(y, \mu^2)$  via the well-known Bethe-Heitler process  $\gamma^* g \rightarrow c\bar{c}$ <sup>5</sup>. A comparison of the various contributions to  $F_2^c$  in NLO shows that for the physically reasonable scales  $\mu$ ,  $\mu \simeq 2m_c \dots \sqrt{Q^2 + 4m_c^2}$  (see below), the quark contribution in (1) — which is not necessarily positive due to mass factorization — is very small, about 5% or less. Therefore  $F_2^c$  does represent a clean gluonic observable also in NLO.

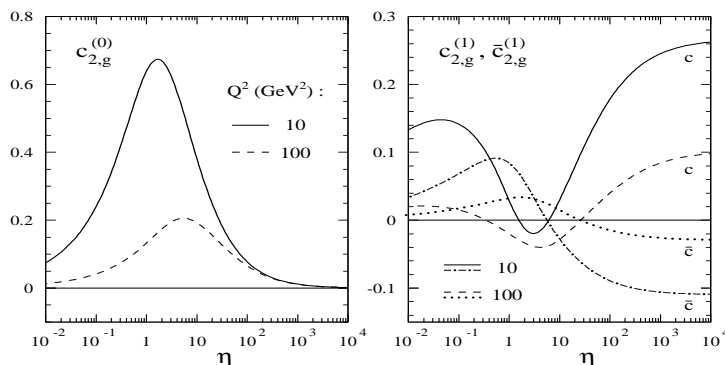


Figure 1: The  $\eta$ -dependence of the coefficient functions  $c_{2,g}^{(0,1)}$ ,  $\bar{c}_{2,g}^{(1)}$  derived in refs.<sup>3–5</sup> for two values of  $Q^2$  with  $m_c = 1.5$  GeV. The scheme-dependent quantity  $c_{2,g}^{(1)}$  is given in the  $\overline{\text{MS}}$  scheme.

The  $\eta$ -dependence of the gluonic coefficient functions<sup>3–5</sup> is recalled in Fig. 1 for two values of  $Q^2$  typical for deep-inelastic small- $x$  studies at HERA, as in the following using  $m_c = 1.5$  GeV. The comparison of the NLO coefficients  $c^{(1)}$  and  $\bar{c}^{(1)}$  with the LO result  $c^{(0)}$  reveals that potentially large corrections arise from regions where  $c^{(0)}$  is small, i.e. from very small and large partonic c.m. energies. These corrections are due to initial-state-gluon bremsstrahlung and the Coulomb singularity at small  $\hat{s}$ , and due to the flavour excitation (FE) process at  $\hat{s} \gg 4m_c^2$  ( $\eta \gg 1$ ). For a more detailed discussion, including the quark contributions, see ref.<sup>3</sup>. Large FE-logarithms have been resummed — at the expense of losing the full small- $\hat{s}$  information of (1) — by introducing a charm parton density, leading to the so-called variable-flavour scheme<sup>6</sup>. For another approach to high  $\hat{s}$  see ref.<sup>7</sup>. The importance of these corrections in the HERA small- $x$  regime considered here will be investigated below.

The first question to be addressed in order to judge the phenomenological usefulness of  $F_2^c$  as a gluon constraint is whether or not the available NLO expression (1) is sufficient for obtaining results which are stable under variation of the (unphysical) mass factorization scale. It has been argued<sup>8</sup> that one

should use  $\mu \simeq 2m_c$ , since  $\mu$  is supposed to be controlled by  $\hat{s}$  and the integrand in (1) is maximal close to the lower limit,  $\hat{s} \simeq 4m_c^2$ . The range of significant contributions in  $\hat{s}$ , however, broadens considerably with increasing  $Q^2$  (see below), hence  $\mu \simeq \sqrt{Q^2 + 4m_c^2}$ , chosen in refs. <sup>1,3,4</sup>, appears at least equally reasonable. Therefore we estimate the theoretical uncertainty of  $F_2^c$  in NLO by varying the scale between  $\mu = (2)m_c$  and  $\mu = 2\sqrt{Q^2 + 4m_c^2}$ , see Fig. 2.

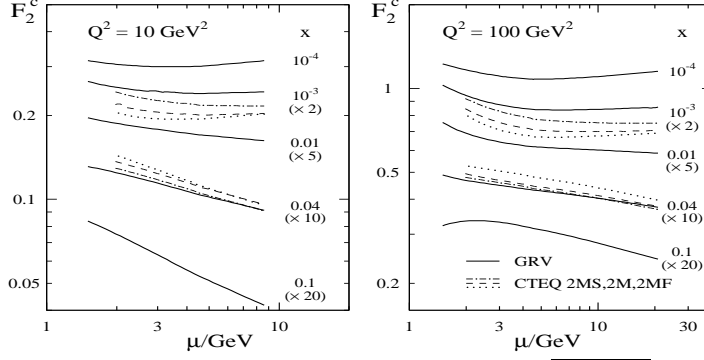


Figure 2: The dependence of  $F_2^c$  on the scale  $\mu$  for  $m_c \leq \mu \leq 2\sqrt{Q^2 + 4m_c^2}$  at selected values of  $x$  and  $Q^2$ . GRV<sup>9</sup> and CTEQ2<sup>10</sup> (at  $x = 10^{-3}, 0.04$ ) parton densities have been employed.

One finds that at small- $x$ ,  $x \lesssim 10^{-2}$ , the scale variation amounts to at most about  $\pm 10\%$ . Moreover, the scale stability at small  $x$  does not significantly depend on the steepness of the gluon distribution. Consequently, the NLO results of ref. <sup>3</sup> seem to provide rather sound a theoretical foundation for a small- $x$  gluon determination at HERA, despite the large total c.m. energy  $s \gg 4m_c^2$  which might suggest a destabilizing importance of  $\ln[\hat{s}/(4m_c^2)]$  terms. At large  $x$ ,  $x \approx 0.1$ , on the other hand, the scale dependence of  $F_2^c$  is rather strong, especially at low  $Q^2$ , presumably due to the large small- $\eta$  threshold contributions mentioned above. However,  $F_2^c$  is small in this region.

The next issue we investigate is the locality in  $y$  of the gluon determination via  $F_2^c$ . The contribution from initial-parton momenta smaller than  $y_{\max}$  to  $F_2^c(x, Q^2)$ , denoted by  $F_2^c(x, Q^2, y_{\max})$ , is presented in Fig. 3 for the GRV parton distributions<sup>9</sup>. At scales  $\mu \approx \sqrt{Q^2 + 4m_c^2}$ , about 80% of  $F_2^c$  originates in the region  $y_{\min} = ax \leq y \lesssim 3y_{\min}$ . Again the situation is very similar for the CTEQ2 parton densities<sup>10</sup>. Thus  $F_2^c$  allows for rather local determination of  $g(y, \mu^2 \approx \sqrt{Q^2 + 4m_c^2})$ . The partonic c.m. energies in the region contributing 80% to the structure function  $F_2^c$  are given by  $\eta \lesssim 5$  (20), corresponding to  $\hat{s} \lesssim 60$  (180) GeV<sup>2</sup>, at  $Q^2 = 10$  (100) GeV<sup>2</sup>, respectively. This implies that for the  $Q^2$  values under consideration here, the plateau region of  $c^{(1)}$  and  $\bar{c}^{(1)}$  at large  $\eta$  (c.f. Fig. 1) does not play an important role. Resummation of large- $\eta$

FE logarithms is thus not necessary, and not appropriate if it implies additional approximations in the more important small- $\hat{s}$  region<sup>6</sup>. A similar observation has already been made in<sup>8</sup> for  $\mu \simeq 2m_c$  at low  $Q^2$ . The latter scale choice leads however to a considerably wider important range of  $\hat{s}$  at high  $Q^2$ , somewhat in contrast to its motivation described above.

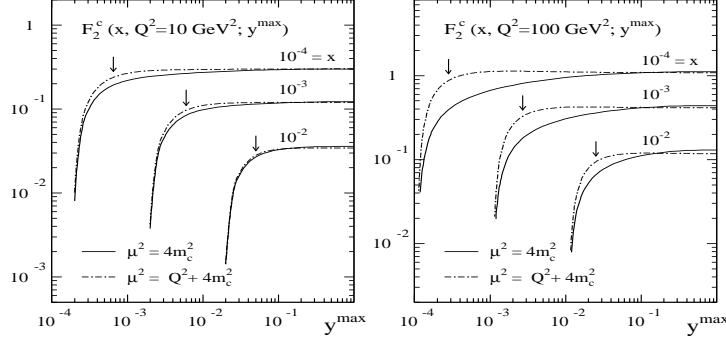


Figure 3: The contribution of the initial-parton momentum region  $ax \leq y \leq y^{\max}$  to  $F_2^c$  at small  $x$  for two choices of the scale  $\mu$ , using the parton densities of ref.<sup>9</sup>. The arrows indicate the values of  $y^{\max}$  at which 80% of the complete results are reached for  $\mu = \sqrt{Q^2 + 4m_c^2}$ .

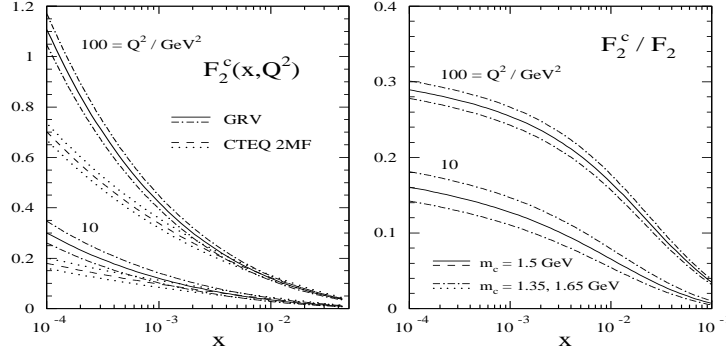


Figure 4: The  $x$ -dependence of  $F_2^c$  and  $F_2^c/F_2$  at two fixed values of  $Q^2$ , as expected from the GRV gluon density<sup>9</sup>. Also shown are  $F_2^c$  as obtained from the CTEQ 2MF parton set<sup>10</sup> and the charm mass dependence of the predictions.  $\mu = \sqrt{Q^2 + 4m_c^2}$  has been employed.

The expected absolute and relative sizes of  $F_2^c$  are displayed in Fig. 4. In contrast to the bottom contribution  $F_2^b$  which reaches at most 2...3%,  $F_2^c$  is large in the HERA small- $x$  region, making up up to a quarter of  $F_2$  as measured at HERA. See also ref.<sup>1</sup>. This size of  $F_2^c$  renders a reliable (fully massive) treatment mandatory in any precise analysis of  $F_2$  at small  $x$ . The sensitivity of  $F_2^c$  to the gluon density is illustrated by the difference of the CTEQ 2MF (flat  $xg(x, \mu^2 = 2.6 \text{ GeV}^2)$ <sup>10</sup> and the GRV (steep gluon)<sup>9</sup> expectations. There

is quite some discriminative power of  $F_2^c$ , especially close to the lower end of the  $Q^2$  range considered here,  $Q^2 \approx 10 \text{ GeV}^2$ . Moreover, by measuring up to about  $100 \text{ GeV}^2$  in  $Q^2$ , the rapid growth of  $yg(y \ll 1, \mu^2)$  predicted by the Altarelli-Parisi equations can be rather directly tested down to  $y \simeq 10^{-3}$ . A theoretical obstacle to an easy accurate gluon determination via the charm structure function is the dependence of  $F_2^c$  on the unknown precise value of the charm quark mass  $m_c$ . A  $\pm 10\%$  variation of  $m_c$  also considered in Fig. 4 leads to a  $\pm 15 \dots 25\%$  ( $5 \dots 10\%$ ) effect at  $Q^2 = 10$  ( $100$ )  $\text{GeV}^2$ , respectively.

To summarize: The NLO perturbative QCD approach to the charm structure function  $F_2^c$  seems to be in good shape at small  $x$  where  $F_2^c$  is expected to be large (up to about a quarter of the total  $F_2$  in the HERA regime), with scale variations of less than about  $\pm 10\%$ . The situation is worse at large  $x$ , where the structure function is however small.  $F_2^c$  represents a clean gluonic observable in NLO and is well suited for rather local a gluon measurement at small- $x$ , with bulk of the result originating from gluon momenta within a factor of three above the threshold value. Flavour excitation contributions from  $\hat{s} \gg 4m_c^2$  become important only at scales  $Q^2$  higher than those relevant for small- $x$  observations at HERA considered here. Despite the significant charm mass dependence of the results, especially at low  $Q^2$ , useful constraints will be put on the proton's gluon density and its evolution at small  $x$ , if  $F_2^c$  data with an accuracy on the 10% level can be obtained at HERA.

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